

# A Homogeneous Non-equilibrium Two-phase Critical Flow Model

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## ABSTRACT

A non-equilibrium two-phase single-component critical (choked) flow model for cryogenic fluids is developed from first principle thermodynamics. Modern equations-of-state (EOS) based upon the Helmholtz free energy concepts are incorporated into the methodology. Extensive validation of the model is provided with the NASA cryogenic data tabulated for hydrogen, methane, nitrogen, and oxygen critical flow experiments performed with four different nozzles. The model is used to develop a hydrogen critical flow map for stagnation states in the liquid and supercritical regions.

## 1.0 INTRODUCTION

The purpose of this report is to formulate a non-equilibrium, two-phase, critical flow model for cryogenic fluids based upon first principal thermodynamics. The model can be used to accurately calculate discharge mass flow rates from high pressure reservoirs.

## 2.0 MODERN EQUATIONS OF STATE

Modern equations-of-state [1] are often formulated using the Helmholtz energy as the fundamental property with independent variables of temperature and density,

$$\alpha(T, \rho) = \alpha^0(T, \rho) + \alpha^r(T, \rho), \quad (1)$$

where  $\alpha$  is the Helmholtz energy,  $\alpha^0(T, \rho)$  is the ideal gas contribution to the Helmholtz energy, and  $\alpha^r(T, \rho)$  is the residual Helmholtz energy, which corresponds to the influence of intermolecular forces in real gases. Thermodynamics properties can be calculated as derivatives of the Helmholtz energy. For example, the pressure can be expressed as

$$p = \rho^2 \left( \frac{\partial \alpha}{\partial \rho} \right)_T. \quad (2)$$

In practical applications, the functional form is explicit in the dimensionless Helmholtz energy,  $\alpha$ , using independent variables of dimensionless density and temperature. The form of this equation is

$$\frac{\alpha(T, \rho)}{RT} = \alpha(\tau, \delta) = \alpha^0(\tau, \delta) + \alpha^r(\tau, \delta) \quad (3)$$

where  $\tau = T_c/T$ , the inverse reduced temperature,  $\delta = \rho/\rho_c$ , the reduced density and  $R$  is the universal gas constant (8.314510 J/(mol\*K)).

The ideal gas Helmholtz energy is often represented in the computational convenient parameterized form

$$\alpha^0(\tau, \delta) = \ln \delta + a_0 \ln \tau + a_1 + a_2 \tau + \sum_{k=3}^N a_k \ln[1 - \exp(b_k \tau)], \quad (4)$$

and the residual contribution to the Helmholtz free energy takes the form

$$\alpha^r(\tau, \delta) = \sum_{i=1}^l N_i \delta^{d_i} \tau^{t_i} + \sum_{i=l+1}^m N_i \delta^{d_i} \tau^{t_i} \exp(-\delta^{p_i}) + \sum_{i=m+1}^n N_i \delta^{d_i} \tau^{t_i} \exp\left[+\varphi_i(\delta - D_i)^2 + \beta_i(\tau - \gamma_i)^2\right], \quad (5)$$

where the parameters and coefficients in these expressions are given for hydrogen (normal, parahydrogen and orthohydrogen) [2], oxygen [3], nitrogen [4], methane [5], and water [6].

The advantages of this explicit formulation in the Helmholtz free energy become apparent for the calculation of enthalpy, entropy, and sound speed, respectively:

$$h(T, \rho) = RT \left\{ \tau \left[ \left( \frac{\partial \alpha^0}{\partial \tau} \right)_{\delta} + \left( \frac{\partial \alpha^r}{\partial \tau} \right)_{\delta} \right] + \delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_{\tau} + 1 \right\}, \quad (6)$$

$$s(T, \rho) = R \left\{ \tau \left[ \left( \frac{\partial \alpha^0}{\partial \tau} \right)_{\delta} + \left( \frac{\partial \alpha^r}{\partial \tau} \right)_{\delta} \right] - \alpha^0 - \alpha^r \right\}, \text{ and} \quad (7)$$

$$w(T, \rho) = \sqrt{\frac{RT}{M} \left[ 1 + 2\delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_{\tau} + \delta^2 \left( \frac{\partial^2 \alpha^r}{\partial \delta^2} \right)_{\tau} - \frac{\left[ 1 + \delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_{\tau} - \delta \tau \left( \frac{\partial^2 \alpha^r}{\partial \delta \partial \tau} \right) \right]^2}{\tau^2 \left[ \left( \frac{\partial^2 \alpha^0}{\partial \tau^2} \right)_{\delta} + \left( \frac{\partial^2 \alpha^r}{\partial \tau^2} \right)_{\delta} \right]} \right]}. \quad (8)$$

Other fluid properties can be found in references [1,7].

The saturation line can be described by the ancillary equation [1] for the saturated vapor-pressure,  $p_{sat}$  as

$$\ln \left( \frac{p_{sat}}{p_c} \right) = \frac{T_c}{T} \sum_{i=1}^n N_i \left( 1 - \frac{T}{T_c} \right)^{k_i} \quad (9)$$

where  $p_c$  is the critical pressure. The derivative of the vapor-pressure, which shall be used later in this report, reads as follows:

$$\frac{dp_{sat}}{dT} = -\frac{p_{sat}}{T} \left[ \ln \left( \frac{p_{sat}}{p_c} \right) + \sum_{i=1}^n k_i N_i \left( 1 - \frac{T}{T_c} \right)^{k_i-1} \right] \quad (10)$$

Table 1 lists the critical constants and molecular weights for each of the substances that are addressed in this report.

Table 1. Critical Properties and Molecular Weights for Hydrogen, Methane, Nitrogen, and Oxygen

Fluid	$P_c$ (MPa)	$T_c$ (K)	$\rho_c$ (kg/m <sup>3</sup> )	M (kg/kmol)
Hydrogen	1.2964	33.145	31.263	2.01588
Methane	4.5992	190.564	162.66	16.0428
Nitrogen	3.3958	126.192	313.300	28.01348
Oxygen	5.043	154.581	436.1	31.9988

### 3.0 CRITICAL DISCHARGE ANALYSIS FROM A HIGH PRESSURE RESERVOIR

#### 3.1 Single-phase choking of a pure substance

The development starts with the differential form of the first law of thermodynamics

$$dh = Tds + vdp, \quad (11)$$

and the control volume form of the conservation of energy

$$h_0 + \frac{1}{2}U_0^2 = h + \frac{1}{2}U^2. \quad (12)$$

Assuming that the process is reversible and adiabatic (an isentropic process with  $ds = 0$ ) then Eq. (11) can be integrated and combined with Eq. (12) to obtain the famous compressible Bernoulli Equation

$$\frac{1}{2}U^2 - \frac{1}{2}U_0^2 = \int_p^{p_0} v \cdot dp \quad (13)$$

The upstream reservoir variables, the stagnation state, where the velocity is often assumed zero at location “0”, at any instant are considered in a quasi-steady-state, and as such, the velocity at the choked or critical location “t” can be expressed in terms of the integral along a streamline outside the boundary layer flow to yield

$$U_t = \left[ 2 \int_{p_t}^{p_0} \frac{1}{\rho} dp \right]^{\frac{1}{2}} \quad (14)$$

At location “t”, the critical discharge mass flux is then

$$G_t = \rho_t U_t = \rho_t \left[ 2 \int_{p_t}^{p_0} \frac{1}{\rho} dp \right]^{\frac{1}{2}}. \quad (15)$$

The task is to find the maximum of this function, that is, to find the pressure,  $p_t$ , such that the mass flux is maximum, which is the definition of the classical critical flow or choked condition, or should the maximum occur at the lowest pressure in the system, the flow is considered subcritical. This

approach is referred to as the “Homogeneous Direct Integration” (HDI) method [8,9]. With the use of the above equation of state, it is straight forward to generate a table of paired pressure-density values from the stagnation state along an isentrope to a pressure less than the choked pressure (for example discharging into the atmosphere at 0.1 MPa) and perform the direct integration of Eq. (15). The maximum value of the integration is found to be the critical discharge mass flux.

An equivalent, but more rigorous methodology, which we name the “Homogeneous Direct Evaluation” (HDE) method, that directly exploits the equations-of-state discussed above, is to consider the energy equation (12) while neglecting the upstream velocity, i.e.,  $U_0 = 0$ ,

$$h_0 = h_t + \frac{U_t^2}{2},$$

which is arranged to the convenient general mass flux form

$$G = \rho \left[ 2(h_0 - h) \right]^{\frac{1}{2}}. \quad (16)$$

As above, critical flow requires a local maximum of Eq. (16), or  $\left( \frac{\partial G}{\partial p} \right)_t = 0$ . When this condition is applied to Eq. (16) one obtains

$$\frac{2(h_0 - h)}{v} = - \frac{\left( \frac{\partial h}{\partial p} \right)_s}{\left( \frac{\partial v}{\partial p} \right)_s}. \quad (17)$$

From the first law of thermodynamics Eq. (1)

$$\left( \frac{\partial h}{\partial p} \right)_s = v, \quad (18)$$

and noting from the definition of sound speed squared

$$w^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = - \frac{v^2}{\left( \frac{\partial v}{\partial p} \right)_s}, \quad (19)$$

one can combine Eqs. (17-19) to get

$$2(h_0 - h_t) = w_t^2 \quad (20)$$

It's not surprising that the maximum velocity of a critical flow condition is the sound speed.

The interesting fact is that by directly solving the coupled isentropic and critical flow conditions

$$\begin{aligned} s_0(T_0, \rho_0) &= s(T_t, \rho_t) \\ 2[h_0(T_0, \rho_0) - h(T_t, \rho_t)] &= w(T_t, \rho_t)^2 \end{aligned} \quad (21)$$

for  $T_t$  and  $\rho_t$ , the exact critical flow state for the given stagnation condition is obtained. The advantages of the HDE method over the HDI method are two: 1. A table of paired density-pressures need not be created, and 2. The local maximum mass flux need not be found using a search technique. The HDE method, by solving system (21), directly determines the exact critical mass flux condition.

In the impressive work at NASA by Simoneau and Hendricks [10], four different nozzles were used to investigate choked flow for a number of cryogenic fluids (hydrogen, nitrogen and methane). Gaseous nitrogen was used to calibrate the four nozzles, and a Table was presented with the results [10]. The same Table is presented here for completeness as well as to compare, in the last two columns, the HDE method, the solution of system (21), with the original Table results.

Table 2. NASA Table [10] for gaseous nitrogen with the proposed HDE model results in bold type added in the last two columns.

Nozzle	Stagnation Temperature (K)	Stagnation pressure (N/cm <sup>2</sup> )	Ratio of throat to stagnation pressure (measured)	Ratio of throat to stagnation pressure (calculated)	Maximum measured mass flux, $G_{meas}$ (g/cm <sup>2</sup> *s)	Maximum calculated mass flux, $G_{calc}$ (g/cm <sup>2</sup> *s)	Mass flux ratio $G_{meas} / G_{calc}$	<b>Proposed HDE model mass flux (g/cm<sup>2</sup>*s)</b>	<b>Proposed HDE model throat to stagnation pressure</b>
7° conical	272	356	0.522	0.524	846	872	0.970	<b>873</b>	<b>0.523</b>
3.5° conical	276.5	351	0.537	0.524	820	852	0.962	<b>853</b>	<b>0.523</b>
2D	284	343	0.565	0.524	790	820	0.963	<b>821</b>	<b>0.523</b>
Elliptical	233	313	0.495	0.524	820	835	0.982	<b>837</b>	<b>0.524</b>

The agreement is excellent between the HDE model and the NASA experiments for both critical mass flux and ratio of throat to stagnation pressure. Note that the mass flux ratio is the effective discharge coefficient for each of the individual nozzles. This discharge coefficient shall be applied in the two-phase analysis described below.

### 3.2 Two-phase choking of a pure substance

The development of the two-phase methodology follows directly from the single-phase approach. There are a number of assumptions that should be noted: 1. The stagnation condition is a pure substance at saturated liquid, subcooled liquid, or supercritical such that an isentropic expansion from the stagnation state to the saturation line, the saturation locus, occurs through the compressed liquid region and not the superheated vapor side of the critical point. This assumption is not very critical to the final results provided the stagnation state is supercritical and not superheated, but since we're mostly interested in the liquid side, we state this condition, 2. The two-phase flow is homogeneous, 3. The two-phase flow is in mechanical equilibrium; that is, the phases have equal velocities. The methodology could be extended into regimes with slip or relative velocity between the phase, but in our direct application (see below), the choked vapor volume fraction is usually less than 10%, so the mechanical coupling between the phases is large; and therefore, relative velocities small, 4. The vapor phase is at saturation, 5. The liquid phase may be in a metastable state (superheated state). 6. The phases share a common pressure (the vapor saturation pressure), 7. The mixture flow is adiabatic and frictionless; and therefore isentropic, and 8. The discharge location has a short L/D ratio. For example, one can imagine a rupture or puncture of a high pressure reservoir wall, or a short nozzle.

The HDE method applied to two-phase critical conditions requires two steps: 1. Expand from the stagnation conditions to the liquid saturation line, the saturation locus, ( $s_s = s_0$ ,  $h_s$ ,  $T_s$ ,  $p_s$ , and  $\rho_s$ ), and 2. Expand from the liquid saturation locus into the two-phase coexistence region.

If the stagnation state is saturated liquid, step 1 is omitted. It remains implicit in this two-step procedure that the maximum liquid superheat allowed for any vapor temperature less than  $T_s$  is that the liquid temperature  $T_l = T_s$ .

The general pure substance two-phase relationships between various fluid properties and the quality,  $x$ , are introduced

$$\begin{aligned} s_{2p} &= x \cdot s(T_v, \rho_v) + (1-x) \cdot s(T_l, \rho_l) = x \cdot s_v + (1-x) \cdot s_l \\ v_{2p} &= x \cdot v(T_v, \rho_v) + (1-x) \cdot v(T_l, \rho_l) = x \cdot v_v + (1-x) \cdot v_l \\ h_{2p} &= x \cdot h(T_v, \rho_v) + (1-x) \cdot h(T_l, \rho_l) = x \cdot h_v + (1-x) \cdot h_l \end{aligned} \quad (22)$$

If one determines the temperature and density for each phase at the choke plane, then the problem is solved. An analysis is presented below to determine those 4 properties.

The two-phase mass flux equation, derived from the conservation of energy, is a direct extension of Eq. (16) with the enthalpy and specific volume states replace with two-phase conditions

$$G = \frac{1}{v_{2p}} \left[ 2(h_0 - h_{2p}) \right]^{\frac{1}{2}} \quad (23)$$

The two-phase extension of system (21) governing the two-phase critical flow requires four equations in the four unknowns  $T_l$ ,  $\rho_l$ ,  $T_v$ , and  $\rho_v$ . This system is: (1) the conservation of energy, (2) the conservation of entropy,  $ds = 0$ , (3) the vapor component is saturated, and (4) both phases share the same pressure. This system is written

$$\begin{aligned} 2(h_0 - h_{2p}) &= w_{2p}^2 \\ s_{2p} &= s_0 \\ p_{sat} &= p_v \\ p_l &= p_v = p \end{aligned} \quad (24)$$

where the squared two-phase sound speed can be written

$$w_{2p}^2 = \left( \frac{\partial p}{\partial \rho_{2p}} \right)_s = - \frac{v_{2p}^2}{\left( \frac{\partial v_{2p}}{\partial p} \right)_s} = - \frac{v_{2p}^2}{x \left( \frac{\partial v_v}{\partial p} \right)_s + (1-x) \left( \frac{\partial v_l}{\partial p} \right)_s + (v_v - v_l) \left( \frac{\partial x}{\partial p} \right)_s} \quad (25)$$

Making use of Eq. (19) for each phase results in

$$w_{2p}^2 = \frac{v_{2p}^2}{\frac{x}{(\rho_v w_v)^2} + \frac{(1-x)}{(\rho_l w_l)^2} - (v_v - v_l) \left( \frac{\partial x}{\partial p} \right)_s} \quad (26)$$

or in terms of the mass flux,  $G = \rho w$ ,

$$G_{2,p}^2 = \frac{1}{\frac{x}{(\rho_v w_v)^2} + \frac{(1-x)}{(\rho_l w_l)^2} - (v_v - v_l) \left( \frac{\partial x}{\partial p} \right)_s}. \quad (27)$$

Should  $\left( \frac{\partial x}{\partial p} \right)_s = 0$ , then the so called “frozen” mass flux or sound speed is defined, which for a homogeneous two-phase mixture in mechanical and thermal equilibrium is the maximum sound speed of the system. The task now is to find the derivative of the quality with respect to pressure holding the system entropy constant. This is accomplished by solving the system entropy in Eq. (22) for the quality

$$x = \frac{s_0 - s(T_l, \rho_l)}{s(T_v, \rho_v) - s(T_l, \rho_l)}, \quad (28)$$

and performing the required differentiation yields

$$\left( \frac{\partial x}{\partial p} \right)_s = \frac{-(s_v - s_l) \left( \frac{\partial s_l}{\partial p} \right)_{s_l} + (s_0 - s_l) \left[ \left( \frac{\partial s_v}{\partial p} \right)_{s_v} - \left( \frac{\partial s_l}{\partial p} \right)_{s_l} \right]}{(s_v - s_l)^2}. \quad (29)$$

Taking  $s_v$  and  $s_l$  to be  $s_v = s_v(T_v, p)$  and  $s_l = s_l(T_l, p)$ , respectively, and then writing the total differentials gives

$$\begin{aligned} \left( \frac{\partial s_v}{\partial p} \right)_{s_v} &= \left( \frac{\partial s_v}{\partial p} \right)_{T_v} \left( \frac{\partial p}{\partial p} \right)_{s_v} + \left( \frac{\partial s_v}{\partial T_v} \right)_p \left( \frac{\partial T_v}{\partial p} \right)_{s_v} \\ \left( \frac{\partial s_l}{\partial p} \right)_{s_l} &= \left( \frac{\partial s_l}{\partial p} \right)_{T_l} \left( \frac{\partial p}{\partial p} \right)_{s_l} + \left( \frac{\partial s_l}{\partial T_l} \right)_p \left( \frac{\partial T_l}{\partial p} \right)_{s_l} \end{aligned} \quad (30)$$

Upon recognizing Maxwell’s fourth relationship

$$\left( \frac{\partial s}{\partial p} \right)_T = - \left( \frac{\partial v}{\partial T} \right)_p = - \frac{\beta}{\rho} \quad (31)$$

and the definition of the specific heat at constant pressure

$$c_p = T \left( \frac{\partial s}{\partial T} \right)_p, \quad (32)$$

one can write Eq. (30)

$$\begin{aligned} \left( \frac{\partial s_v}{\partial p} \right)_{s_v} &= - \frac{\beta_v}{\rho_v} + \left( \frac{c_{pv}}{T_v} \right) \left( \frac{\partial T_v}{\partial p} \right)_{s_v} \\ \left( \frac{\partial s_l}{\partial p} \right)_{s_l} &= - \frac{\beta_l}{\rho_l} + \left( \frac{c_{pl}}{T_l} \right)_p \left( \frac{\partial T_l}{\partial p} \right)_{s_l} \end{aligned} \quad (33)$$

Note that the isobaric heat capacity,  $c_p$ , and the volume expansivity,  $\beta$ , can be directly calculated from the EOS in section 2, respectively, as

$$c_p(T, \rho) = c_v + R \frac{\left[ 1 + \delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_\tau - \delta \tau \left( \frac{\partial^2 \alpha^r}{\partial \delta \partial \tau} \right) \right]^2}{\left[ 1 + 2\delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_\tau + \delta^2 \left( \frac{\partial^2 \alpha^r}{\partial \delta^2} \right)_\tau \right]} \quad \text{and} \quad \beta(T, \rho) = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p = \frac{1}{\rho} \left( \frac{\partial p}{\partial T} \right)_\rho \left( \frac{\partial \rho}{\partial p} \right)_T$$

where

$$\left( \frac{\partial p}{\partial \rho} \right)_T = RT \left[ 1 + 2\delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_\tau + \delta^2 \left( \frac{\partial^2 \alpha^r}{\partial \delta^2} \right)_\tau \right] \quad \text{and} \quad \left( \frac{\partial p}{\partial T} \right)_\rho = R\rho \left[ 1 + \delta \left( \frac{\partial \alpha^r}{\partial \delta} \right)_\tau - \delta \tau \left( \frac{\partial^2 \alpha^r}{\partial \delta \partial \tau} \right) \right].$$

Relating the liquid and vapor temperatures, by defining a “non-equilibrium” parameter,  $\eta$ , can be defined in the following manner:

$$T_l = \eta \cdot T_s + (1 - \eta) \cdot T_v. \quad (34)$$

As stated above, the maximum thermal non-equilibrium liquid superheat allowed is  $T_l = T_s$  (the saturation locus from step 1 where the fluid is expanded to the liquid saturation line) when  $\eta = 1$ , and least superheat is  $T_l = T_v$  when  $\eta = 0$ . The latter case,  $\eta = 0$ , defaults to the well know Homogeneous Equilibrium Model (HEM), that is with the mixture in both thermal and mechanical equilibrium. This analysis provides all degrees of liquid superheat, from none, the HEM, to liquid temperatures at the saturation locus. Principally because of Eq. (34) we’ve restricted, by assumption 1 of the model, to the liquid side of the critical point; otherwise, an assumption concerning metastable vapor, supercooled vapor, would be necessary, and where vapor volume fractions become greater than 0.5, mechanical equilibrium may not be valid as the vapor can accelerate more quickly than the liquid droplets.

Differentiating Eq. (34) where the pressure is only a function of temperature on the saturation line, Eq. (10), Eq. (33) for the two distinct phases becomes

$$\left( \frac{\partial s_v}{\partial p} \right)_{s_v} = -\frac{\beta(T_v, \rho_v)}{\rho_v} + \frac{\left( \frac{c_p(T_v, \rho_v)}{T_v} \right)}{\left( \frac{dp_\sigma(T_v)}{dT_v} \right)} \quad (35)$$

$$\left( \frac{\partial s_l}{\partial p} \right)_{s_l} = -\frac{\beta(T_l, \rho_l)}{\rho_l} + (1 - \eta) \frac{\left( \frac{c_p(T_l, \rho_l)}{T_l} \right)_p}{\left( \frac{dp_\sigma(T_v)}{dT_v} \right)}$$

The sound speed (26), or the mass flux based sound speed (27), can be computed knowing the four phasic unknowns of temperatures and densities along with Eqs (28), (29), (34) and (35). The system of equations (24) is closed, and the details are reviewed here



$$\begin{aligned}
2[h_0 - h_{2p}(x, T_l, \rho_l, T_v, \rho_v)] &= w_{2p}(\eta, x, T_l, \rho_l, T_v, \rho_v)^2 \\
s_{2p}(x, T_l, \rho_l, T_v, \rho_v) &= s_0 \\
p_{sat}(T_v) &= p_v(T_v, \rho_v) \\
p_l(T_l, \rho_l) &= p_v(T_v, \rho_v) \\
x &= \frac{s_0 - s_l(T_l, \rho_l)}{s_v(T_v, \rho_v) - s_l(T_l, \rho_l)} \\
\eta &= \frac{T_s - T_v}{T_l - T_v}
\end{aligned} \tag{36}$$

Note that the last two expressions are only used for convenience since they are not independent relationships; and therefore are already in terms of the four unknown variables.

#### 4.0 HDE MODEL VALIDATION

The NASA cryogenic critical flow data [10,11] was used to validate the non-equilibrium, two-phase, critical flow model described by the system (36). The results are shown for hydrogen [10], methane [10], nitrogen [10,11], and oxygen [11], respectively, in Figures 1-4. The calculated values have been corrected with the discharge coefficient, the mass flux ratio, given in Table 2. In each Figure, a T-S diagram insert is included to display the analyzed stagnation conditions. The computed results appear to be consistently greater than the measured mass fluxes; but in the overall, the solution of system (36) provides very good agreement with the experimental data.

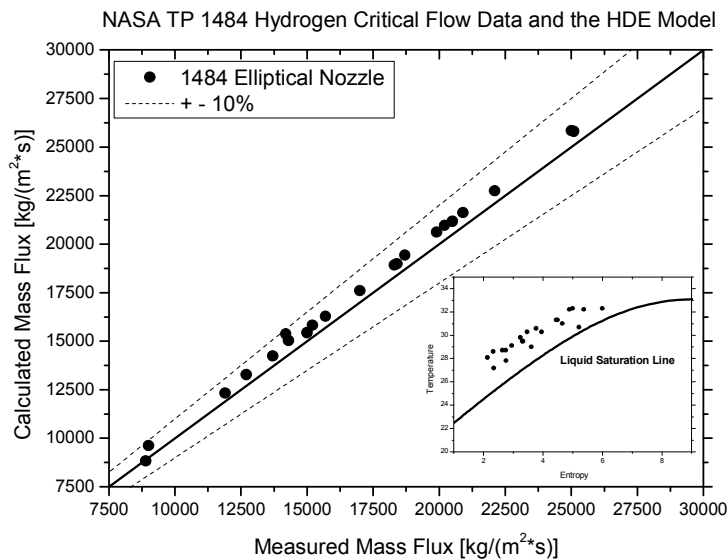


Figure 1. HDE calculated critical mass fluxes and the NASA hydrogen data [10]

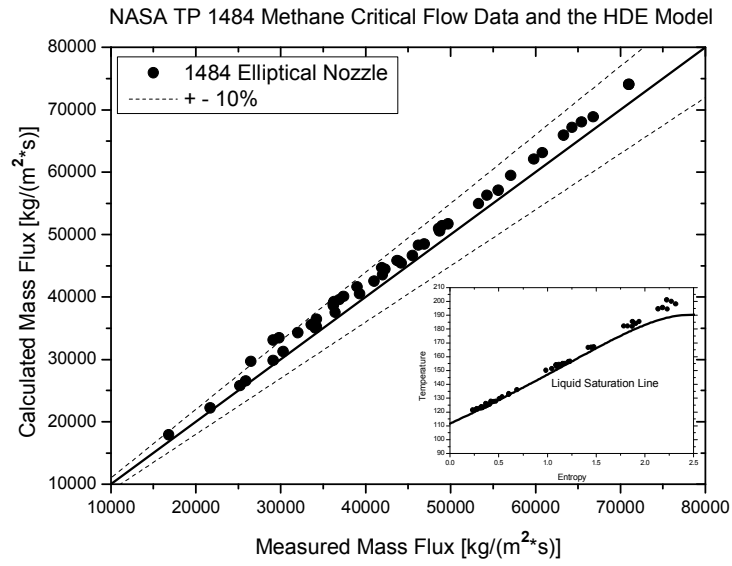


Figure 2. HDE calculated critical mass fluxes and the NASA methane data [10]

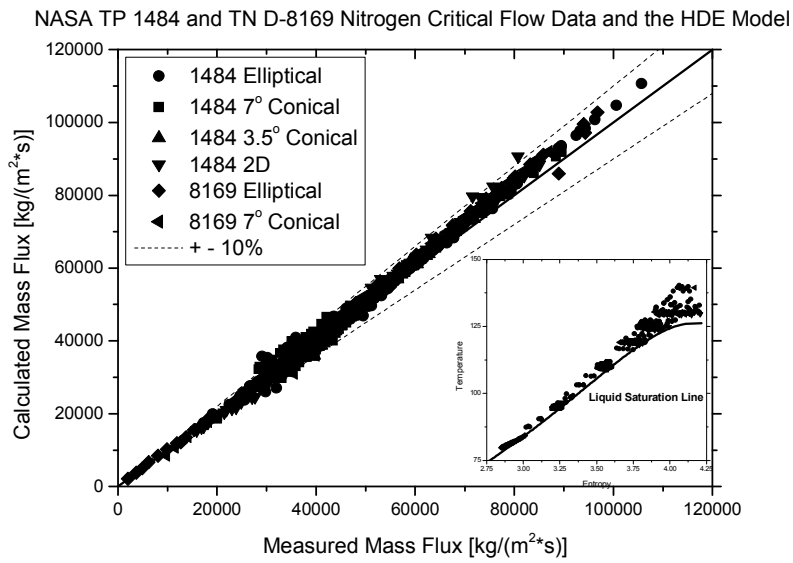


Figure 3. HDE calculated critical mass fluxes and the NASA nitrogen data [10, 11]

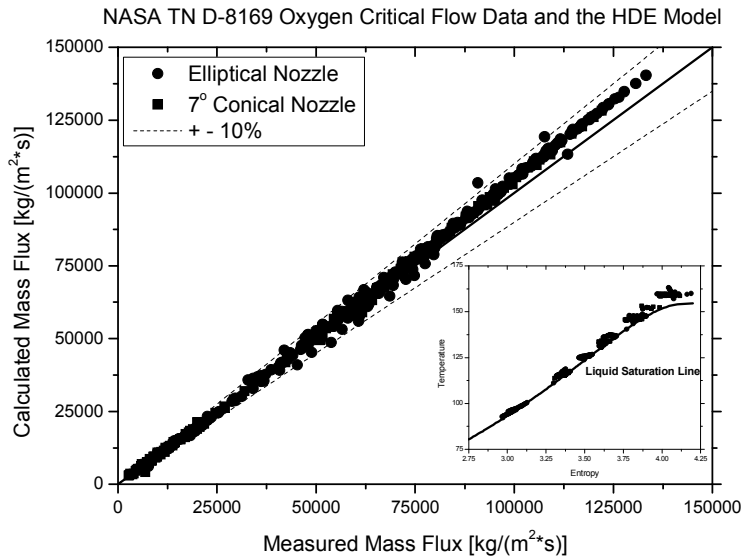


Figure 4. HDE calculated critical mass fluxes and the NASA oxygen data [11]

### 5.0 HDE MODEL CALCULATED HYDROGEN CRITICAL MASS FLUXES

The HDE model was used to develop a critical flow map for liquid and supercritical hydrogen. Stagnation conditions are shown in the inserted hydrogen T-S diagram (Figure 5), where the stagnation temperature,  $26K \leq T_0 \leq 40K$ , and pressure,  $P_0 \leq 6MPa$ , states are always in the single phase region with entropy,  $S_0 \leq S_{critical}$ . After determining the mass flux from the critical flow map in Figure 5, one should correct it with the relevant discharge coefficient.

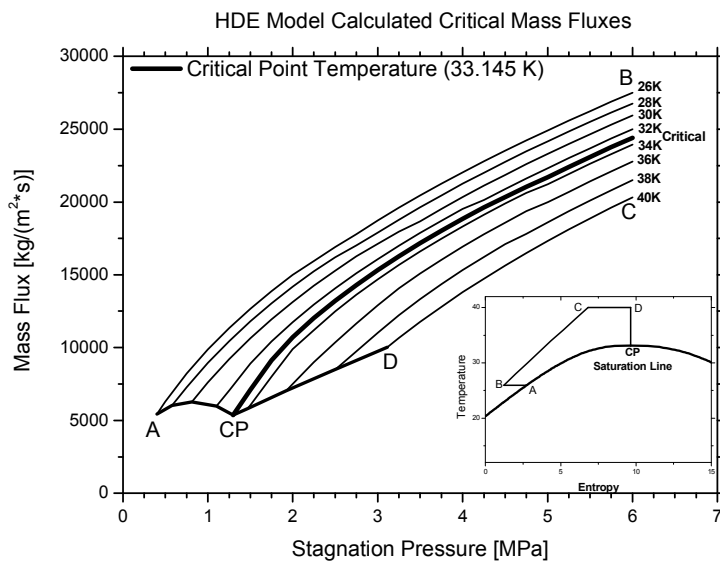


Figure 5. HDE calculated critical mass fluxes for hydrogen with stagnation states in the liquid and supercritical regions

## 6.0 CONCLUSIONS

A homogeneous non-equilibrium, two-phase, critical flow model, the homogeneous direct evaluation model (HDE), has been developed from first principal thermodynamics and modern equation-of-state formulations. The model has been validated with extensive cryogenic data involving liquid and supercritical hydrogen, methane, nitrogen, and oxygen. A critical discharge flow map for hydrogen is presented that allows the reader a straightforward procedure to determine critical mass fluxes for a range of stagnation conditions.

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