ANALYSIS OF THE PARAMETRIC-ACOUSTIC INSTABILITY FOR SAFETY ASSESSMENT OF HYDROGEN-AIR MIXTURES IN CLOSED VOLUMES

Yáñez, J., Kuznetsov, M., Redlinger, R., Kotchourko, A., Lelyakin, A.,
Institute for Energy and Nuclear Technique, Karlsruhe Institute of Technology, Kaiserstraße 12, 76131 Karlsruhe, Germany. Email jorge.yanez@kit.edu

ABSTRACT
The acoustic to the parametric instability has been studied for H₂-air mixtures at normal conditions. Two approaches for the investigation of the problem have been considered. The simplified analytical model proposed by Bychkov was selected initially. Its range of applicability resulted to be very restricted, and therefore, numerical solutions of the problem were taken into account. The results obtained were used to study the existence of spontaneous transition from the acoustic to the parametric instability for different fuel concentrations. Finally, the growth rate of the instabilities was numerically calculated for a set of typical mixtures for hydrogen safety.

NOMENCLATURE LISTING

<table>
<thead>
<tr>
<th>A</th>
<th>Auxiliary variable</th>
<th>Le</th>
<th>Lewis number</th>
<th>a</th>
<th>Auxiliary variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Auxiliary variable</td>
<td>Ma</td>
<td>Markstein number</td>
<td>β</td>
<td>Auxiliary variable</td>
</tr>
<tr>
<td>B</td>
<td>Auxiliary variable</td>
<td>Pr</td>
<td>Prandtl number</td>
<td>γ</td>
<td>Auxiliary variable</td>
</tr>
<tr>
<td>C₁</td>
<td>Auxiliary variable</td>
<td>q</td>
<td>Auxiliary variable</td>
<td>δ</td>
<td>Auxiliary variable</td>
</tr>
<tr>
<td>C₂</td>
<td>Auxiliary variable</td>
<td>Tₜ</td>
<td>Temp. comb. products</td>
<td>θ</td>
<td>Dimensionless temp.</td>
</tr>
<tr>
<td>Cᵢ</td>
<td>Heat at constant pressure</td>
<td>Tᵢ</td>
<td>Temp. reactants</td>
<td>Θ</td>
<td>Expansion ratio</td>
</tr>
<tr>
<td>F</td>
<td>Flame front</td>
<td>Uₐ</td>
<td>Vel. of the acoustic field</td>
<td>κ</td>
<td>Auxiliary variable</td>
</tr>
<tr>
<td>gₑ</td>
<td>Acceleration of gravity</td>
<td>Uₙ</td>
<td>Laminar flame velocity</td>
<td>λ</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>H</td>
<td>Auxiliary variable</td>
<td>Y</td>
<td>Auxiliary variable</td>
<td>ρ</td>
<td>density</td>
</tr>
<tr>
<td>I</td>
<td>Auxiliary variable</td>
<td>z</td>
<td>Auxiliary variable</td>
<td>σ</td>
<td>Growth rate</td>
</tr>
<tr>
<td>J</td>
<td>Auxiliary variable</td>
<td>Zₑ</td>
<td>Zeldovich number</td>
<td>χ</td>
<td>thermal diffusivity</td>
</tr>
<tr>
<td>k</td>
<td>Wavenumber</td>
<td></td>
<td></td>
<td>ω</td>
<td>Frequency acoustic field</td>
</tr>
<tr>
<td>L</td>
<td>Laminar flame thickness</td>
<td></td>
<td></td>
<td>ψ</td>
<td>Auxiliary variable</td>
</tr>
</tbody>
</table>

1 - INTRODUCTION

Generation of pressure waves by a flame in closed volumes is well known phenomenon. The interaction between those waves and the surface of the flame is a feedback process in which pressure wave intensity and heat released by the flame influence each other. Markstein [1] concluded that the coupling between the both phenomena was due to the variation of the flame surface produced by the mechanism described here. The alternative velocity field created by the pressure waves produces oscillations of the amplitude of the cellular structures existing in the flames. This variation of the surface alters, in turn, the total amount of fuel consumed and the heat released by the flame.

Two different instabilities due to flame - pressure waves interaction have been identified [2]. In the acoustic instability (see [3]) the cellular structures of the flame front oscillate with the frequency of the acoustic alternative field. Two effects tend to muffle it. For large wavenumbers the instability is damped by diffusive processes. For small wavenumbers it is absorbed by the effect of gravity. The acoustic instability corresponds, for zero amplitude of the excitation velocity, to the Darrieus-Landau planar instability. For increased values of the alternative velocity [3], the acoustic instability has the notable property of being able to stabilize the Darrieus-Landau instability.
The acoustic instability is followed, for enhanced alternative velocity, by the parametric one. Under the later the growth rate is generally superior than in the acoustic case. The cellular structures of the flame oscillate with a frequency half of the acoustic, fact that was recognized by Markstein as the typical signal of the Kapitsa parametrically dumped pendulum, who consequently named so the instability.

From the point of view of the prediction of the severity of an explosion, the gaseous mixtures can be classified in two groups [5]. If the two instabilities co-exist for some ranges of acoustic velocities, a planar flame front is never stable and the acoustic instability transform spontaneously into the parametric. If they do not co-exist for any range of acoustic velocities, the acoustic instability tends to suppress the Darrieus-Landau instability, the parametric instability regime is never reached and planar flame fronts are stable as long as the alternative velocity field exists.

These two differentiate propagation regimes have been confirmed by the observations of Searby [8] and Aldredge and Killingsworth [7] who performed experiments with downwards propagating flames inside a cylindrical and an annular burner respectively. It was found that the flame propagation was divided in four differentiate stages. Just after ignition, the flame surface quickly became wrinkled due to the Darrieus-Landau instability and as the flame propagates further sound was generated. Due to the fundamental acoustic instability, these waves caused an attenuation of flame wrinkles. The Darrieus-Landau instability was suppressed and the flame became planar. Depending on whether the parametric and the acoustic instability coexist or not, the secondary parametric instability developed producing significant flame acceleration and the appearance of large organized pulsating cellular structures. In the final stage of development those coherent cellular structures were converted in incoherent flame surfaces fluctuations.

Therefore, gaseous mixtures prone to the parametric instability may suffer, in closed chambers, a very significant acceleration of the flame propagation velocity. Especially for lean mixtures, this increase of the combustion rate will be very considerable.

The mathematical treatment of the acoustic and parametric instabilities is based on the work of Pelce and Clavin [4], who obtained an equation for a perturbed flame front in a gravitational field. Based on those results, Searby and Rochwerger [5] managed to derive equations for the growth rate of the acoustic and parametric instability and were able to calculate the stability limits for both cases numerically. Some years later, Bychkov [6] simplified the previous formulation for the cases of small perturbation of a flame front and was able to obtain an analytical solution.

In the present study, the mentioned methodologies were applied to find out which mixtures tend to the parametric instability and which ones not under the typical conditions for hydrogen safety. The ranges of frequency and amplitude of the acoustic waves where the instability exists were identified for different hydrogen concentrations under normal pressure and temperature.

2 – ANALYSIS

The stability of a gaseous mixture with respect to the acoustic and the parametric instability can be calculated utilizing the methodology contained in the references [6], [5]. Let the flame front be represented by the function $F(x,t)=0$ in a reference commoving with the flame front. The small perturbations of the front could be considered in the form $F(x,t)=F(t)e^{i(kx)}$. Considering the problem linear, the second order differential equation (1) describes the evolution of small amplitude flame surface perturbations considering periodic velocity fluctuations normal to the flame front

$$A \frac{d^2 F}{dt^2} + U_e \frac{d F}{dt} + k B \frac{d F}{dt} + k g_a C_i F - k \omega U_e \cos(\omega t) C_i F + U_e^2 k^2 C_i F = 0.$$  

(1)

Where,
\[ A = \left(1 + \frac{\theta - 1}{\theta + 1} kL \left(Ma - \frac{\theta}{\theta - 1} J \right) \right), \quad B = \left(\frac{2\theta}{\theta + 1} \left(1 + \theta kL (Ma - J) \right) \right), \] (2)

\[ C_1 = \left(\frac{\theta - 1}{\theta + 1} \left(1 - kL \left(Ma - \frac{J\theta}{\theta - 1} \right) \right) \right), \] (3)

\[ C_2 = \left(\frac{\theta - 1}{\theta + 1} \left(-1 + \frac{kL}{\theta - 1} \left((3\theta - 1)Ma - 2J\theta + 2Prh_b(\theta - 1) - I(2Pr - 1) \right) \right) \right), \] (4)

\[ \mathcal{G} = (T - T_u) / (T_b - T_u), \quad \gamma = (\rho_a - \rho_b) / \rho_a, \quad h(\mathcal{G}) = \frac{\lambda(\mathcal{G}) C_a}{\lambda_a C(\mathcal{G})} = \frac{\rho(\mathcal{G}) \chi(\mathcal{G})}{\rho_a \chi_a}, \] (5)

\[ H = \int_0^1 (h_b - h(\mathcal{G})) d\mathcal{G}, \quad J = \frac{\gamma}{1 - \gamma} \int_0^1 \frac{h(\mathcal{G})}{1 + \mathcal{G} \gamma / (1 - \gamma)} d\mathcal{G}, \quad I = (\theta - 1) \int_0^1 h(\mathcal{G}) d\mathcal{G}, \] (6)

\[ Ma = \frac{J}{\gamma} - \frac{1}{2} Z e(Le - 1) \int_0^1 \frac{h(\mathcal{G}) \ln(\mathcal{G})}{1 + \mathcal{G} \gamma / (1 - \gamma)} d\mathcal{G}. \] (7)

A very easy simplification may help to understand the meaning of the concept of stability. In the case \( g_a = 0 \) and \( U_a = 0 \), the temporal part of the solution of equation (1) has the form \( F(t) = Y \exp(\sigma t) \) being \( Y \) a constant. Therefore, the planar flame front will be stable with respect to perturbations for all growth rates fulfilling \( Re(\sigma) < 0 \) and unstable otherwise. In the more complex case under study, the solution is more complicated, see [12] (for Bychkov analysis \( f(t) = f_1(t) + f_2 \cos(\omega t) \)) and [5] (see eq. (17)), but the concept remains.

2.1-Bychkov analysis

The analytical solutions obtained by Bychkov [12] were derived in the limit of high acoustic frequency and long wavelength flame surface perturbations. We will study whether the conditions to be fulfilled for their applicability are soundly satisfied.

2.1.1-Acoustic instability

In the case of the acoustic instability, for problems in which it is fulfilled that

\[ \left| \frac{C_i U_{ik}^2 k^2}{Ao^2} \right| \ll 1, \] (8)

\[ \left| \frac{C_i U_{ik}}{2 Ao} \right| \ll 1, \] (9)

\[ \left| \frac{C_i g_{ik} k}{Ao^2} \right| \ll 1, \] (10)

the growth rate admits an explicit solution [6], and \( \sigma \) is directly,
\[
\sigma = -\frac{B U_1 k'}{2A} \pm \frac{1}{2A} \sqrt{\left(\frac{B U_1 k'}{2A}\right)^2 - \frac{1}{A} \left(C_i k g + C_2 U_1^2 k^2 + \frac{C_1^2 U_2^2 k^2}{2A}\right)}.
\]  

(11)

Additionally, the border delimiting the instability region could be easily obtained,

\[
\frac{U_s^2}{U_i^2} = \frac{-2Ag}{C_i k' U_i^2} - 2\frac{C_i}{C_i} A.
\]  

(12)

2.1.2-Parametric instability

Analogously, in the cases that

\[
\frac{|CU_k|}{A\omega} \ll 1,
\]  

(13)

the growth rate for the parametric instability can be calculated solving the implicit equation

\[
\left(A \sigma^2 - A \left(\frac{\omega}{2}\right)^2 + U_1 kB\sigma + kgC_1 + U_2^2 kC_2\right)^2 - \left(\frac{1}{2} \omega^2 U_s C_1\right)^2 + \left(A\sigma\omega + \frac{1}{2} U_1 kB\omega\right)^2 = 0.
\]  

(14)

And the line describing the border of the stable region is

\[
2^2 k^2 \omega^2 C_i^2 U_i^2 \left(-A \frac{\omega^2}{2^2} + kgC_1 + U_2^2 kC_2\right)^2 + B^2 = \left(\frac{U_s}{U_i}\right)^2.
\]  

(15)

2.1.3- Particularization for H₂-Air mixtures in normal conditions

The equations appearing in previous sections were particularized for H₂-air mixtures at normal conditions. For every fuel concentration, the value of the molecular transport coefficient was obtained utilizing the molecular theory of gases [13]. The thermodynamic data was acquired using the JANAF tables [14]. The Lewis number of the mixture was calculated by means of the formulation proposed by Sun et al. [15]. Special care was taken to calculate the overall activation energy. The detailed chemistry scheme of Lutz [16] was coupled with the computer program Cantera [17] to obtain the induction time of the analyzed mixtures. This magnitude was then utilized to derive the overall activation energy. The frequency of the acoustic perturbation \( \omega \), is a parameter which depends on the geometry of the particular problem to be addressed. The range of sizes of containers, tubes, deposits, etc. that practical safety assessments must cover is very wide. Typical examples would be from the analysis of combustion tubes (~0.1 m of radius) to nuclear power plants (~ 25 m radius). For the general purpose of our analysis, excitation frequencies from \( \omega = 20 \) to 4000 Hz could be considered, which respectively correspond, for stoichiometric mixtures, to containers, buildings, etc. of 20, to 0.1 m of radius for the first harmonic.

The particularization of the equations (8-10) and (13) for the whole range of concentrations is represented in Figure 1. The curves relative to the equations (8-10) represent the maximum value for the left hand side (rhs) of those equations in the whole range of wavenumbers in which the instability takes place. The curve corresponding to the equation (13) describes the maximum values of its left hand side in the analyzed interval of wavenumbers (0-4000 m⁻¹). Out of the interval considered, the left hand side of the equation (13) is going to be even bigger, which substantiate the unusual shape of this curve. As can be seen in the figure, the values obtained for the left hand side of the equations (8) (9) and (13) for the H₂-air mixture under analysis are of the order of magnitude of 1. Therefore, the assumptions considered by Bychkov in his analysis are not fulfilled for our particular problem.
2.2-Numerical stability analysis

The equation (1) may be transformed for a more convenient treatment. The change of variables \( \alpha = A, \beta = U, kB, \psi = kC, C_1 + U^2 kC_2, \delta = k\omega C_1, \) allow writing the equation (1) as

\[
\alpha \frac{d^2 F}{dt^2} + \beta \frac{dF}{dt} + \left( \psi - \delta \cos(\omega t) \right) = 0. \tag{16}
\]

Which solution, as stated in [5], is of the kind

\[
F = Y(z)e^{-xz}e^{ky}, \tag{17}
\]

where the new variables are defined as, \( z = 2\omega t, \kappa = \beta \omega \alpha, a = 4\alpha \psi - \beta^2 \omega \alpha \) and \( q = 2\delta \omega \alpha. \) The solution (17) is then substituted in (16). A much simpler differential equation for the variable \( Y \) appears,

\[
Y^\prime\prime + (a - 2q) \cos(2z) Y = 0, \tag{18}
\]

which is known as the Mathieu equation.

2.2.1-Mathieu equation

An extensive analysis of the solutions of the Mathieu equation may be found in [9]. Contrary to the procedure used by Searby and Rochwerger in [5], the Whittaker’s method [10] was selected to obtain the solutions. It considers a solution for the Mathieu equation of the type,

\[
Y(z) = e^{\nu z} \sum_{k=-\infty}^{\infty} c_{2k} e^{i2kz}. \tag{19}
\]

Substituting this in (18), operating (details can be found in the references [12] and [11]) it is found that to be a solution, the variable \( V \) should fulfill the condition
\[
\cos(\pi \nu) = 1 - \Delta(0, a, q)\left[1 - \cos(\pi \sqrt{a})\right]
\]

where

\[
\Delta(\nu, a, q, k) = \begin{vmatrix}
1 & \gamma_{2k} & \ldots \\
\gamma_{2(k-1)} & 1 & \gamma_{2(k-1)} & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
& & & 1 & \gamma_{2(k-1)} & \ldots \\
\end{vmatrix}
\]

with \(\gamma_{2k} = q(2k - \nu)^2 - a\). The Sträng method \[11\] allow calculating \(\Delta(0, a, q, k)\) with the recursion formula,

\[
\Delta(0, a, q, k) = \beta_{2k}\Delta(0, a, q, k - 1) - \alpha_{2k}\beta_{2k}\Delta(0, a, q, k - 2) + \alpha_{2k}\alpha_{2(k-1)}^2\Delta(0, a, q, k - 3),
\]

with \(\alpha_{2k} = \gamma_{2k}\gamma_{2(k-1)}\) and \(\beta_{2k} = 1 - \alpha_{2k}\) and \(q, a \in \mathbb{R}\). This implies that \(\gamma_{2i} \in \mathbb{R}\) \(\forall i \in \mathbb{N}\), so that \(\Delta(\nu, a, q, k) \in \mathbb{R}\). Additionally, \(a \in \mathbb{R}\) and so

\[
\cos(\pi \sqrt{a}) = \begin{cases} 
\cos(\pi \sqrt{a}) & a \geq 0 \\
\cosh(\pi \sqrt{a}) & a < 0 
\end{cases}
\]

is always real. We may nevertheless study the solutions of \(\nu \in \mathbb{C}, \quad \nu = u + wi, \quad u, w \in \mathbb{R}\). Thus,

\[
\cos(i(u + wi)) = \cos(\pi u)\cosh(\pi w) - i\sin(\pi u)\sinh(\pi w)
\]

and then

\[
\cos(\pi u)\cosh(\pi w) - i\sin(\pi u)\sinh(\pi w) = 1 - \Delta(0, a, q)\left[1 - \cos(\pi \sqrt{a})\right].
\]

From the previous considerations,

\[
\cos(\pi u)\cosh(\pi w) = 1 - \Delta(0, a, q)\left[1 - \cos(\pi \sqrt{a})\right],
\]

\(sin(\pi u)\sinh(\pi w) = 0\). \hspace{1cm} (26)

The solutions of (26) are \(u = 0 + k\pi\); or \(w = 0\). So, \(\nu\) is purely real or complex. Therefore, the solutions of equation (25) are, if \(w = 0\)

\[
u = \frac{1}{\pi} \arccos\left[1 - \Delta(0, a, q)\left[1 - \cos(\pi \sqrt{a})\right]\right],
\]

and if \(u = 0 + k\pi\),

\[
\text{(27)}
\]
\[ \cosh(\pi w) = \pm \left\{ 1 - \Delta(0, a, q) \left[ 1 - \cos(\pi \sqrt{a}) \right] \right\}, \]  

(28)

and so,

\[ w = \pm \frac{1}{\pi} \arccosh \left\{ 1 - \Delta(0, a, q) \left[ 1 - \cos(\pi \sqrt{a}) \right] \right\}. \]  

(29)

Thus, if \[ \left| 1 - \Delta(0, a, q) \left[ 1 - \cos(\pi \sqrt{a}) \right] \right| \leq 1 \], \( \nu \) (see eq. (19)) is real, and otherwise complex. Recalling the definition of the solution,

\[ Y(z) = e^{i(u + iw)z} \sum_{k=-\infty}^{\infty} c_{2k} e^{2kz} = e^{-iw} e^{i\omega z} \sum_{k=-\infty}^{\infty} c_{2k} e^{2kz}. \]  

(30)

Substituting (19) in (17)

\[ F = e^{-w} e^{iky} Y(z) = e^{-iw} e^{i\omega z} e^{-w} e^{i\omega k} \sum_{k=-\infty}^{\infty} c_{2k} e^{2kz} = e^{(\omega - \kappa)z} e^{i(\omega z + ky)} \sum_{k=-\infty}^{\infty} c_{2k} e^{2kz}, \]  

(31)

that is stable in the case \(- \nu < \kappa\). Due to the plus-minus solution of \( w \) eq. (29) it must happen that \( \left| \nu \right| < \kappa \) and so,

\[ \frac{1}{\pi} \arccosh \left\{ 1 - \Delta(0, a, q) \left[ 1 - \cos(\pi \sqrt{a}) \right] \right\} < \frac{B}{\omega \alpha}. \]  

(32)

And additional and very interesting conclusion is that, for the cases \( \kappa < 0 \), the stability is impossible.

2.2.2-Negative \( \kappa \)

The definition of \( \kappa \) was \( \kappa = \frac{U_kkB}{\omega A} \). The variables \( U_k, k \) and \( \omega \) are always positive. \( A \), and \( B \) eq. (2) can be in principle positive or negative depending on the values of \( Ma, J, \theta, L, \) and \( k \). The Markstein number, because of the nature of the integrand can be written as

\[ Ma = \frac{\theta}{\theta - 1} J + \frac{1}{2} Ze(Le - 1) \left[ \int_0^1 \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right]. \]  

(33)

Applying (33)

\[ A = \left( 1 + \frac{\theta - 1}{\theta + 1} kL + \frac{1}{2} Ze(Le - 1) \left[ \int_0^1 \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right] \right), \]  

(34)

\[ B = \frac{2\theta}{\theta + 1} kL \left( \int_0^1 \frac{h(\theta)}{1 + \theta(\theta - 1)} \, d\theta + \frac{1}{2} Ze(Le - 1) \left[ \int_0^1 \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right] \right). \]  

(35)

Defining \( k_{0,4} \) and \( k_{0,8} \) as the wavenumbers in which \( A \) and \( B \) change of sign (the condition \( k_{0,4} = 0 \) represents a resonance)
\[
k_{0A} = \frac{-1}{\theta - L -1 \left( \frac{1}{2} Ze(L - 1) \left[ \int_{0}^{1} \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right] \right)},
\]
(36)

\[
k_{0B} = \frac{-1}{\theta L \left( \int_{0}^{1} \frac{h(\theta)}{1 + \theta(\theta - 1)} \, d\theta + \frac{1}{2} Ze(L - 1) \left[ \int_{0}^{1} \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right] \right)},
\]
(37)

Remark that \( Le - 1 \) is smaller than zero in case \( Le < 1 \). For those cases, for \( k > k_{0A} \) the coefficient \( A \) is negative, being

\[
k_{0A} = -10 - L \left( \frac{1}{2} Ze(L - 1) \left[ \int_{0}^{1} \frac{h(\theta) \ln(\theta)}{1 + \theta(\theta - 1)} \, d\theta \right] \right).
\]
(49)

In the case that both \( k_{0A}, k_{0B} \in \mathbb{R}^+ \) there will exist a band of wavenumbers, if we suppose \( k_{0A} < k_{0B} \) for wavenumbers \( k \) that are \( 0 < k_{0A} < k < k_{0B} < +\infty \), in which the flame is completely unstable. In the case only one of the values \( k_{0A} \in \mathbb{R}^+ \) or \( k_{0B} \in \mathbb{R}^+ \) then the band will extend to the infinite. Note additionally that this interval \( 0 < k_{0A} < k < k_{0B} < +\infty \) is independent of the value of the excitation frequency \( \omega \).

The solutions in which the variable \( \kappa \) is negative represent a significant instability mechanism. The gaseous mixtures in which \( \kappa < 0 \) result to be unstable for all intensities of the amplitude of the velocity perturbation for wavenumbers inside the range \([k_{0A}, k_{0B}]\). Due to this characteristic, in the opinion of the authors, this mechanism may be very prone to couple with the perturbations produced by waves, reflections, etc., in practical applications. Additionally, \( \kappa \) depend inversely on the frequency of the perturbation. Therefore, the greatest impact of the mentioned coupling mechanism would be for lean mixtures enclosed in big vessels.

![Figure 2. Bands of complete instability for different concentrations for hydrogen air mixtures in normal conditions.](image)

2.2.3- Particularization for H₂-Air mixtures in normal conditions

The Figure 2 contains the particularization of the variables \( k_{0A}, k_{0B} \) for H₂-air mixtures. It represent the limits in which the effect described in the previous section exists. For fuel concentrations superior to the 30% vol H₂, those mixtures are free from the existence of resonance. For lean mixtures, the band
on instability is centered in relatively low wavenumbers, fact that enhances the implication of the effect (see also Figure 3 upper right).

The Figure 3 contains the results of the application of the method for an excitation of 1000 Hz and normal conditions for fuel concentrations of 7.5, 12.5, 15., 30., 45., 60.% vol. H₂. The diagrams represent the growth rate in a colour scale for different combinations of flame surface wavenumbers (abscissa) and reduced velocities (ordinate). Only positive growth rates are plotted. The stable regions (pairs \((k, U_a/S_L)\) in which the growth rate \(\sigma\) is less or equal than 0) are plotted in violet. Black lines separate the stable regions from the unstable.

Figure 3. Stability graphs for H₂-air mixtures at normal conditions. Excitation frequency of 1000 Hz. From left to right and from top to bottom 7.5, 12.5, 15, 30, 45, 60 vol. H₂.
In the diagram corresponding to 7.5% it is represented the situation in which both $k_{0.0} = 0$ (left) and $k_{0.4} = 0$ (right, resonance) lines are visible (vertical lines). The stripe which lies between the lines is unstable. Left of this band appears a wavenumber range in which no positive growth rates are detected for the interval of reduced velocities considered. In the figure corresponding to 12.5 % H$_2$ only a small part of the $k_{0.0} = 0$ line is visible. The resonance lays in higher wavenumbers than the ones shown. Additionally, the acoustic instability (bottom left corner, still with a low growth rate) and the parametric instability (left) are clearly identifiable. The plots corresponding to 15% and 30% vol. H$_2$ show the appearance of the acoustic instability region with a growing intensity. Moreover, they depict a reduction of the overlap between the acoustic and the parametric instability. In the picture related to 45% vol. H$_2$, an horizontal stripe free of instability appears for intensities $U_a/S_i$ of around 4.5. This band grows with an increased fuel concentration as it is shown in the plot for 60% vol. H$_2$. The absence of overlapping between both instabilities is a very interesting phenomenon. It not only prevents the spontaneous developing of the acoustic instability in the parametric but tends to the suppression of the DL instability (as higher $U_a/S_i$ values produce negative growth rates).

Therefore, the thickness of the stripe free of any instability was investigated for perturbations of different frequencies and concentrations. The results obtained are plotted in the Figure 4. Some interesting conclusions can be obtained. Firstly, an enlarged perturbation frequency increases the thickness of the stability band. Also, higher perturbation frequencies enlarge the interval of concentrations in which the acoustic instability will not transform spontaneously into the parametric. In the extreme case of 20. Hz perturbations, no concentration interval resulted to be stable. This could be significant for the nuclear safety, in which the contention building, due to its size, will produce such order of magnitude of frequencies.

The frequency of the perturbation also changes the whole structure of the instability region. The Figure 5 helps to understand the interesting and complex effect of the acoustic frequency. The stability diagrams for 20% vol. H$_2$-air mixtures at normal conditions are depicted for six different frequencies of 20., 100., 200., 600., 1000., 4000. Hz. The diagram corresponding to 20. Hz shows a complex overlapping between the acoustic and the parametric instability. Additionally, shows that the different leaves of the parametric instability come one after another without any stability gap. For higher frequencies, 100. Hz and more, there is a fine gap of stable region between the parametric and the acoustic instability meaning that two differentiate regions already exist. A further increase of the frequency will cause the displacement of the lobe corresponding to the parametric instability to higher wavenumbers. This displacement is very intense for the highest frequency considered of 4000. Hz (~ 10. cm wavelength for first harmonic in stoichiometric conditions).

Figure 4. Amplitude of the stability band between the acoustic and the parametric instability.
CONCLUSIONS

The analytical solutions obtained by Bychkov, due to the assumptions considered in its obtaining, resulted to be not applicable for H₂-air mixtures problems at normal conditions. The numerical analysis must then be used. The Mathieu differential equation, resulting from the derivation of this methodology, can be efficiently solved utilizing the Whittaker’s method and the Sträng’s formula. Utilizing this methodology, the stability of H₂-air mixtures at normal conditions was studied. It was shown that for concentrations leaner than 30% vol. H₂ the acoustic and the parametric instability always superposed. For richer mixtures, a stable gap between them that prevents a spontaneous transition from the former to the later appeared. The analysis was also performed for different excitation frequencies. It was found that, for small frequencies, the transition to parametric instability was unavoidable as the parametric and the acoustic instability superimposed in an intricate way. It
was ascertained that the thickness of the stability band increased with higher excitation frequencies. The results obtained show a very important dependency on concentration and excitation frequency that could be visualized in the diagrams plotted.

REFERENCES

11. Sträng, J., E., On the characteristics exponents of Fouquet solutions to the Mathieu equation. ULM-TP/05-4