THE CORRELATION METHOD TO ANALYZE THE GAS MIXING PROCESS ON THE BASIS OF BOS METHOD

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ABSTRACT

Structures formed during gas mixing following an injection of a gas into atmosphere are analyzed using optic methods based on the detection of density non-uniformities. Methods for determination of fractal parameters for a random distribution of these non-uniformities are described, and information revealed on the gas mixing structure is analyzed. The BOS (background oriented schlieren) technique is utilized to obtain the optical image of the forming structures, which afterward is processed using the correlation procedure, allowing to extract the quantitative information on the mixing. Additionally, a possibility to link the characteristics of the injected gas source and the system fractal parameters was demonstrated. The method can be used in the development of the non-contact methods for the evaluation of the gaseous system parameters based on the optical diagnostics and, potentially, for the obtaining more detailed information of the gaseous turbulence.

1.0. INTRODUCTION

The object of the current study is the structures formed during the mixing process of two gases going along the release of a gas in a motionless gaseous medium with the discrepant density. Such release typically leads to the turbulent character of the mixing process [1 - 3], i.e., the process builds up by interaction of vortices of various size and energy transfer from larger vortices to the smaller ones. The turbulence physically is multilateral event which includes various phenomena [4 - 6], while the current interest is limited to the geometric properties of the object structures, which can be discovered by a visualization technique. Additionally, a possibility to obtain quantitative information characterizing the process and allowing to link the source of an injected gas, both as a function of the distance from the source and using the temporal dependences, was analyzed.

The analogy of structures appearing in the course of gas mixing with those for other similar phenomena [7 - 12] as, for instance, a combustion, a motion of liquid and disperse mixtures, atmospheric and oceanic phenomena, etc., is evident. Therefore, as a basis for the current study, the known surveys of the complex structures for other similar phenomena were taken with the goal to collect the basic concepts developed for other processes which can be used for the structures under consideration.

In the examination of the turbulent gas motion, the random structures resulted from the interaction of the vortices of various sizes should be taken into account. Versatile information can be obtained exploring them from the standpoint of information that can be obtained from the geometrical analysis of these structures. Separating them between certain size scales, the parameters, which are actually the information about the structure, are introduced. Then, the random structures in small scale range are characterized by fractal properties and can be described by several parameters including the fractal dimension and scales of small and large distances by which fractal properties are restricted.

In the current study the smallest scale is not related to the physical nature of the structures under consideration rather is determined by the resolution of the devices used together with the detection method. Hence, only two parameters of the fractal structure of the analyzed random system, namely, the fractal dimension of the structure *D* and the correlation length R^* characterizing the scale at which the distribution of the perturbations becomes uniform, are expected. At larger scales the structure should remain almost uniform for distances *r* in a range $R^* < r < R_{max}$, and the parameter R_{max} then characterize a size that is occupied by vortices in this system. Thus, the structures resulted from the

mixing of gases can be characterized by three parameters D, R^* , R_{max} , while the sizes R^* and R_{max} can be different depending on the distance from the source of gas injection. In the following sections the description of the developed method for determination of these parameters and the discussion on the possibility to link these parameters and the injection source characteristics are presented.

2.0. DESCRIPTION OF THE METHOD

2.1. Theoretical background of the method

We use the standard BOS method [16 - 18] to obtain an optical picture resulted from gas mixing and combine it with the correlation method for the processing of the optical picture as it is used for the analysis of fractal structures. Starting from Mandelbrot [13] who explained the nature of fractal structures, the fractal concepts are used in physics of random systems and processes widely. Below, we use standard fractal methods for the analysis of the structures with geometrical restrictions. On this way, we firstly consider a fractal aggregate as a visual demonstration of geometric structures formed as a result of joining of identical particles, say, form association of spherical particles of a radius *a*. The fractal aggregate is a random rare structure which parameters depend on the character of joining of elemental particles. Let us divide a volume occupied by the fractal aggregate under consideration in *N* identical volumes and construct the correlation function

$$\mathcal{C}(\mathbf{r}) = \frac{1}{N} \sum_{i} \rho(\mathbf{r}_{i}) \rho(\mathbf{r}_{i} + \mathbf{r}) = \frac{\langle \rho(\mathbf{r}_{i}) \rho(\mathbf{r}_{i} + \mathbf{r}) \rangle}{\langle \rho(\mathbf{r}_{i}) \rangle},\tag{1}$$

where *i* is a number of an element, $\rho(\mathbf{r}_i)$ is the density at the location of an element *i*, so that this value is one for an occupied element and zero for an empty element. This correlation function depends on a distance *r* between points as

$$C(r) = \frac{Const}{r^{d-D}},\tag{2}$$

where d is the dimension of the space under consideration, D is the fractal dimension, the characteristic of this geometric figure. In particular, for metal fractal aggregates resulted from relaxation of vapors of some metal vapors, as it follows from the analysis of a two-dimensional photography of these fractal aggregates, the fractal dimension is close to 1.7 [9, 10], and this corresponds to the mechanism of relaxation [11 - 13] when small hard particles are joined in fractal aggregates, and a typical size of fractal aggregates increases in the course of their evolution.

The developed turbulence is a random structure with vortices of various sizes. Interaction between vortices leads to a certain correlation between velocities at points located on a certain distance [1, 2]. Along this, the variation in the gas density can be used for the analysis of gas turbulent motion. From this standpoint a mixing of gases can be analyzed and thus help to characterize it by particular space parameters which may be measured. Among these parameters, the fractal dimension of a structure is a latent parameter that adds information about the formed structure. For the gaseous systems the fractal dimension of the developed turbulence can be expected to be in the range 1.2 - 1.36.

In the following sections the numerical and experimental technique is described which was used for the data processing. In the first subsection we present the stages of data processing together with the details of numerical implementation this procedure. In the second subsection we introduce the experimental data used for fractal analysis and corresponding discussions.

2.2. Description of the algorithm

The described below technique was intended to build the correlation function C(r) and to obtain characteristic values of the fractal structure of the object under consideration using its BOS image. The characteristics which are expected to be restored from the object image include minimal and maximal sizes of fractal aggregate R^* and R_{max} and the object fractal dimension D.

The principle of the BOS technique is based on the registration of the variations of refractive index connected with the medium density gradients [16 - 18]. For this two images of a deliberately structured background (usually a randomly scattered dot pattern) are prepared and used for processing. The first image is obtained from undisturbed medium and the second one from the medium with the object of interest. The *difference* between these two images provides information about the object. The field gradients in the optical path of imaging rays cause deflection of the light rays, leading to the shift in the image details and consequently providing information on the density perturbations connected with the existing flow turbulence in the object. Displacements are calculated from the images using PIV cross-correlation algorithm.

Thus, the original BOS images are actually the two-dimensional arrays of data corresponding to the integral of the density gradients in the perturbed medium along the optical path from the BOS background to the camera for each position in the image. The values in this array are continuously distributed between null values and their maximum. Building of the correlation function (1) using continuous density distribution $\rho(r_i)$ has known difficulties [19, 20], therefore to avoid them, a stepwise (or threshold) proxy of the original continuous function was introduced. Natural choice in the case of symmetrically scattered values for normalized distribution (values are distributed from 0 to 1) would be discrimination between upper and lower values by the threshold value equal to 0.5, i.e., a grey image pixel is replaced by black pixel for its values less than 0.5 and by white pixel for the values from 0.5 to 1.0.

Note, that the freedom in the definition of the threshold value, on one hand, provides additional capabilities to visually evaluate the quality of the obtained fractal object but, on the other hand, since the selection of the threshold value is subjective, it requires an understanding of the sensitivity of obtained fractal aggregate characteristic to the way of its selection. Another specific attribute of the BOS visualization typically is simultaneous existence of the regions with quite different characteristics of the turbulence. For instance, in a picture with expanding jet the turbulent characteristics of jet obviously changes when moving downward from jet origin. Therefore selection of the region for the evaluation of the fractal characteristic can also influence the results.



Figure 1. Example of fractal correlation function F(r) with the dependence described by (3) obtained from the BOS image.

In the frames of the proposed technique, it is assumed that for the building of the required correlation function, related to the certain area at the image, first, a corresponding rectangular area and the threshold value are selected and then obtained two-dimensional array is processed for reconstruction of the such correlation function. For the convenience of the reconstruction procedure the original correlation function is replaced by the modified F(r)

 $F(r) = \pi r^2 C(r).$

The algorithm of fractal correlation function (FCF) evaluation can be formalized using the following procedure:

- minimal analyzed object is pixel, each element in two-dimensional array of reduced data is called pixel (1 is *white* pixel and 0 is *black* pixel) and is presented in the visualization as a pixel of *white* or *black* color;
- for each *white* pixel a primitive correlation function $C'(x_0, y_0, r)$ is evaluated with the center at this pixel with coordinates (x_0, y_0) ;
- *black* pixels does not contribute into primitive correlation function;
- primitive correlation function *C*'(*x*₀, *y*₀, *r*) is calculated as a number of *white* pixels in a circle of radius of *r* pixels with the center at (*x*₀, *y*₀);
- aggregate FCF C(r) and F(r) are calculated as sum of all primitive correlation functions $C'(x_0, y_0, r)$ normalized by area of the object under consideration.

Since the evaluation of FCF is extremely expensive process the algorithm should offer an option to use only sub-set of entire data, e.g., each 2^{nd} , 4^{th} , 8^{th} , etc pixel from original data. This gives a possibility to make rough estimation of the fractal characteristic in shorter time. Average decrease of the amount of calculations for double pixel reduction is about order of magnitude, i.e., ~10 time less. The amount of calculations depends strongly on the *density* of the original picture, in other words on the amount of *white* pixels. Fortunately, the spectral characteristics are practically insensitive to the reduction rank of data sub-set, thus providing a possibility to evaluate FCF using considerably smaller sub-sets.

Typical correlation function obtained from the BOS photograph (see Figure 1) has three clearly defined regions: growing part from 0 to R^* , constant part from R^* to R_{max} , and decreasing part above R_{max} . The values of R^* and R_{max} can be easily estimated visually from the plot of FCF distribution. In these measurements the parameter R^* may be connected with a used device and characterizes the possibilities of measurement rather than the nature of the process. In the region of sizes between R^* and R_{max} the mixture of gases is more or less uniformly. On the periphery the gas structure has fractal properties, so that the parameters R_{max} and D characterize the nature of gas mixing.

Consider the properties of the obtained correlation function (3).Since the function is obtained from the photography, the dimension of the object is reduced to the two-dimensional case d = 2. As it is mentioned above the range of sizes in the analyzed object can be divided into two parts: a range $a < r < R^*$, where the fractal properties of the structure are valid and essential, and a range $R^* < r < R_{max}$, where the structure is uniform. This behavior is similar to the properties of the aerogel structure [21], where, if R^* is the correlation radius, the FCF on the distances above R^* gives after averaging an identical matter density. Thus, the target correlation function F(r) obtained from the treatment of the BOS image can be approximated by the following dependence

$$F(r) = \left(\frac{r}{a}\right)^{D}, a < r < R^{*},$$
(4)

$$F(r) = Const, R^* < r < R_{max}.$$
(5)

Such schematic dependence of the correlation function on the distance corresponds to that shown in the Figure 1. Along with this, the correlation function with further increase of r decreases in a range $R_{max} < r < R_o$, where the structure is still detected. If the clipping area at the photograph is larger than the object, the value R_{max} can be considered as a transversal size of this structure and is characterized by the possibility of the vortices to drift in a surrounding space. Expression (4) includes a minimum size of the fractal structure a, which is determined by the resolution of the detector. Therefore, it can be taken equal to a unity a = 1 and corresponding to an image pixel in the considered case of photography. The correlation function F(r) is also measured in pixels, thus it starts from F = 1 pixel for R = a.

2.3. Influence of data processing technique details on FCF

A priori the importance of the method application details discussed in the previous section was not known, therefore, the possible effect of the data processing procedure was evaluated. Among all probable sources the following were selected for more detailed examination:

- size of interrogation area in BOS processing;
- shift step value in BOS processing (smaller step corresponds to the higher detailization of the resulting image);
- sub-set rank value for FCF evaluation (selection of each 2nd, 4th, 8th, etc pixel from original image);
- color *threshold* value FCF evaluation;
- extent and position of processed area for FCF evaluation.

2.3.1. BOS-processing parameters

The BOS processing was carried out with variable size of interrogation area; namely 8, 16, and 32 pixels; and variable size of shift step, namely 1, 2, and 4 pixels, while FCF evaluation was made with the sub-set rank value equals to 8, color *threshold* was taken to be equal to average level of *intensity of BOS signal*.

Analyzed data convincingly demonstrated that BOS processing conditions (interrogations area size and shift step) do not have any noticeable influence on resulted value of fractal dimension. This fact is of key importance for the following handling of the data, since it appears to be almost independent on the quality and details of acquiring of BOS photographs. For the first evaluation of new cases, the original images can processed with roughed BOS parameters thus considerably speedup the procedure. Since calculation time in case of IA (interrogation area) equal to 16 pixels and shift step of 8 pixels is about 20 times less than for case with IA equals to 8 pixels and step equals to1 pixel, the gain can be significant.

2.3.2. Subset rank value and color 'threshold' value for FCF evaluation

For clarification of the influence of the data completeness on the fractal dimension, the FCF evaluation was made using various sub-set reduction rank value: 2, 4, 8 and 16. Obtained data showed that value of sub-set rank have little or no influence on the values of $.R^*$, R_{max} , and D.

In the examined cases the connection between value of color *threshold* and received fractal dimension is noticeable. With the view to assure repeatable evaluation of FCF, the same procedure of color *threshold* determination was used in all treatments: an averaged BOS signal from the whole area of the object under consideration was used as separating value.

For determination of possible boundaries for color *threshold*, a series of calculations with variations of its value was performed. It was found that the best results are achieved when the color *threshold* remains within 5% from the average of *BOS signal*.

2.3.3 Size and position of processed area in FCF evaluation

For the determination of the local fractal characteristics of the object the different clipping areas should be used. An evaluation of the influence of the size and position of the clipping area on the results was performed. The obtained results showed that clipping has to be focused exactly on the studied object. The method fails if too small parts of the image are used (less than 5%) or if irrelevant or inappropriate objects are also included in the processed area.

3. RESULTS AND DISCUSSION

The main goal of the experimental part of the work is the study of possibility to provide data of fractal dimension for different gas object using the proposed method and to obtain quantitative information on the fractal characteristics of such objects. Three different sets of experiments were performed:

- the experiment with hydrogen under-expanded jet with the view to obtain information on the fractal characteristics for the developing turbulence along the jet length;
- two experiments with hydrogen plume with the view to obtain fractal characteristics of the flow under conditions of the natural and forced ventilation;
- four experiments with hydrogen jets aimed to establish the possible link between fractal characteristics and the jet source.

In the following subsections we illustrate fractal characteristics of the mixing gases on the basis of the performed experiments to find connection between object properties and fractal dimensions. For each experiment from 10 to 25 images were processed, therefore the results are statistically significant and independent on the possible shortcomings of each individual image. The obtained results are also discussed from the point of view of their sensitivity to the details of the processing method.

3.1. Hydrogen under-expanded jet

In the first case the vertical hydrogen jet was investigated. The hydrogen was released from gas cylinder using pressure control unit through bypass configuration. The hydrogen is supplied by 120 cm long pipe. The tube was connected to an orifice with diameter of 21 mm. The experiment was performed with constant temperature about 290 K and pressure about 10 Bar in the cylinder. The inner dimensions of the experimental test chamber are large enough compared to jet region and have no influence on its properties. Obtained jet images and BOS image have dimensions 4368×2912 pixels and give information on jet dispersion in the area about 1.50×1 m.



Figure 2. Original BOS image (left) and the prepared b/w image (inverted) with the threshold equal to the average BOS signal (right) for cases with different positions of the clipping areas. Red rectangle shows the global extents in which the individual clipping areas were selected.

The first test of the proposed technique was to apply it to the process of turbulence development in the under-expanded jet. Fractal dimension was calculated for different areas of the jet located along the axis at the distances from 2.5 cm up to 152.2 cm from the jet origin with each one equals to 15 cm in height.



Figure 3. Correlation functions for cases with different position of clipping areas. Numbers of the curves correspond to the clipping area elevation listed in Table 1.

The images and the FCF reconstruction are shown in the Figure 2 and Figure 3 and obtained results are summarized in Table 1. The obtained results demonstrated that the image of the jet can be clearly separated into two parts: the initial part of the jet where fractal dimension is in the range 1.6 - 1.3 and the upper part of the jet where the dimension remains in the range 1.3 - 1.2. This separation can be interpreted as differentiation between the jet zone with the developing turbulence and the zone where turbulence has already established character. The distance from jet orifice, where developed turbulence starts, is located about 50 cm downstream for this case. This result is in a very good agreement with the experimental observations and can be considered as a strong argument in support of the technique applicability.

Table 1. Summary of the fractal dimension for cases with different position of clipping areas.

L*, cm	2.5	17.5	32.5	47.5	62.5	77.5	92.5	107.5	122.5	137.5
D**	1.59	1.45	1.34	1.31	1.27	1.24	1.25	1.24	1.22	1.20
\mathbf{Y} distance from the interval in the lower description of the dimension of the height even lie 15 mm.										

*L – distance from the jet orifice to the lower edge of the clipping area with the height equal to 15 cm; **D – fractal dimension.

3.2. Hydrogen plume

In these experiments a process of gas mixing released from medium scale enclosure under conditions of natural and/or forced ventilation was studied. The experimental model was fabricated from metal frames with *Plexiglas* faces to allow optical observation. It had the dimensions $80 \times 70 \times 100$ cm and was equipped with two vent openings with a size of 50×10 cm.

The openings were located close to the bottom and the top of sidewalls on opposite sides. In the considered cases a passive venting and active venting through the upper opening was examined. For the case with active venting two additional fans with the size $80 \times 80 \times 25$ mm and an average air flow of 67.5 m³/h were mounted to the middle of the upper vent opening. In internal volume of the enclosure two obstacles were located. A solid cube with a size $60 \times 50 \times 40$ cm on the bottom and a grid cube with the same size on the top of cell were situated. The orifice was positioned 47.5 cm above

the ground and at the distance of 0.75 cm from the edge of the enclosure. Hydrogen was released horizontally into the enclosure trough the nozzle with internal diameter 8 mm. In these cases a constant mass flow rate of hydrogen was provided equal to 4 g/s.

The examples of the processed images and of the correlation functions are presented in the Figure 4 and Figure 5 and in the Table 2.



Figure 4. Original BOS image (left) and the prepared b/w image (right) for the case with natural ventilation. Red rectangle shows the extents in which the fractal parameters were determined.



Figure 5. Original BOS image (left) and the prepared b/w image (right) for the case with forced ventilation. Red rectangle shows the extents in which the fractal parameters were determined.

The fractal dimensionalities obtained for the plumes resulted from natural and forced ventilation are listed in the Table 2 and demonstrated that only in the case of forced ventilation the fractal dimension corresponds to the values typical for the developed turbulence with self-similar internal structure.

Table 2. Fractal dimension for the vented plume.

	Fractal dimension
Natural ventilation	1.16
Forced ventilation	1.20

3.3. Link between fractal characteristic of the jet and jet source

With the goal to find out possible relations between the fractal characteristics and the conditions of the jet formation, four experiments with the different release orifice and different release pressure were carried out.



Figure 6. The example of the original BOS image (left) and the prepared b/w image (inverted) for the cases with different release pressure and orifice diameter. Red rectangle shows the extents in which the fractal parameters were determined.

In this set of experiments in contrast to the first set of tests, characteristics of horizontal hydrogen jet were studied. From the gas cylinder the hydrogen was supplied through 60 mm long pipe with inner diameter of 10 mm and then was released through an orifice of diameter 1 mm or 4 mm. The orifice is positioned 90 cm above the ground to avoid wall effects. Horizontal orientation of the orifice is adjusted by laser alignment. Pressure of supplied gas during the experiment was maintained constant at the level of 10 or 20 bar with temperature ranged between 290 and 298 K.

The photographs were made in such a way to preserve relative ratio of the area occupied by jet to the whole image; the longitudinal distances for all images in all four tests remained equal. The clipping area used for the evaluation of the correlation function was situated at the same distance relative to the jet origin and was covering identical jet regions.

In the Figure 6 one of the original BOS image and corresponding preparation for FCF evaluation are shown. The values of the obtained fractal dimensionalities are summarized in the Table 3.

Table 3. Fractal dimension for the cases with different release pressure and orifice diameter.

	Orifice Ø1 mm	Orifice Ø4 mm
Release pressure 10 bar	1.27	1.20
Release pressure 20 bar	1.29	1.22



Figure 7. Correlation functions for the cases with different release pressure and orifice diameter. 1 - \emptyset 4 mm, P = 10 bar; 2 - \emptyset 4 mm, P = 20 bar; 3 - \emptyset 1 mm, P = 10 bar; 4 - \emptyset 1 mm, P = 20 bar.

3.4. Discussion

In this section a short discussion on the results of the measurements and the data processing are presented. All the results are related to the mixing processes of hydrogen in air and can be possibly used for the characterization of the process parameters. As one can see from the Tables 1-3 and the plots for correlation function in the Figures 3 and 7, the outcome for different cases varies in the limited range of parameters. The fractal dimension of structures formed during the gas mixing remains within interval of 1.1-1.6. The parameters R^* , R_{max} characterize the rate of mixing and depends both on the distance from the source and the intensity of injected fluxes.

The performed analysis reveals the fractal nature for the boundary of gas mixing in the interface regions between gases. The structure of these peripheral regions has an analogy with the boundary of clouds which fractal dimension is determined on the basis of reflection of electromagnetic waves in radar measurements and is 1.36 ± 0.10 [14, 15]. In the case of the under-expanded jet propagation the proposed method gives the values of fractal dimension constantly decreasing with the increasing of the distance from the orifice. At the distances from 0 to approximately 50 cm it changes from 1.6 to 1.3. These values exceed those predicted by the theory; this fact can be interpreted as a drawback of the method due to the deficiency of the required image resolution and/or as a detection of the transition regime of the turbulence development. At the distances larger than 50 cm, the fractal dimensionalities reduces from 1.3 to 1.2, thus confirming both the ability of the technique to obtain physically relevant

quantitative information and the existence of the regime of the developed turbulence. Note, that at maximum distance the value of the fractal dimension reaches its lower limiting quantity of 1.2, revealing, in our opinion, start of the turbulence degradation.

For the experiments with the vented plume the obtained values of the fractal dimension are near the lower limit. In the case of the natural ventilation it has the value of 1.16, while for the forced ventilation it increases and reaches value of 1.2. The characteristics and the images for the vented plumes and for the upper part of the jet are very similar, and, as it was mentioned above, can be connected with distortion of the turbulent structures.

In the experiments with the variations of the jet source the results demonstrated unambiguous dependence of the fractal dimension on the jet parameters. Clear tendency for its increase with the increase of the pressure and with the decrease of the release orifice is found. Such dependencies can be used in the development of the non-contact methods for the evaluation of the gaseous system parameters based on the optical diagnostics.

In general, the mixing of gases occurs as a result of turbulent motion of an injected gas and during it the energy transports from the larger to the smaller vortices [1-3], which are identified in the analyzed images. Their correlation length R^* characterizes the distance at which the distribution of the perturbation becomes uniform up to R_{max} , and in the range of distances between R_{max} and R_o this perturbation decays. R_o is a distance in the transversal direction that characterizes the transversal propagation of vortices in the course of the drift of the gas release. Note that if the motion of a turbulent jet from the orifice can be assumed having some drift velocity [4-7], then the distance R_o is proportional to the time of the vortex diffusion in the transversal direction during this drift, and using different regimes of gas mixing, potentially, one can determine four characteristic times of the turbulent mixing regime, namely, the drift time, the diffusion time, the time of energy transfer from large scale to small scale vortices, and the diffusion time for large vortices. This requires the additional analysis, but one can in this way to determine the parameters of the source of an adding gas from the parameters of structures that are resulted from gas mixing.

SUMMARY

New correlation method for the analysis of the turbulent gas mixing based on fractal concept combined with the background oriented schlieren technique is proposed. Three sets of experiments on the under-expanded hydrogen jet and hydrogen vented plume propagation in air were carried out and analyzed using proposed method. The obtained results demonstrated that the method can provide statistically significant information on the characteristics of the analyzed systems, in particular for the turbulent gas transport. The method can be used in the development of the non-contact methods for the evaluation of the gaseous system parameters based on the optical diagnostics and, potentially, for the obtaining more detailed information of the gaseous turbulence. Note, that the results can depend on the details of the algorithm, therefore an additional analysis is required with the view to determine both the accuracy of procedure within the framework of a certain algorithm implementation and the accuracy of the specific model used for the description of the structures under consideration.

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